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# Effect of Corruption A Mathematical Study Using Difference Equations

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***Dedicated to Prof. Hari M. Srivastava on his*** $75^{th}$ ***birth anniversary***

**Abstract:** Corruption has been prevalent since ages. Government of all countries are affected by corruption. The purpose of this paper is to analyse effect of corruption in government through expenditure and revenue. Mathematical study is done by using difference equations. There is a consensus that real magnitude of corruption can not be measured. In this paper, the corruption is being estimated by a parameter corruption index of the country under consideration.

The aim of this article is to create awareness on the effect of corruption on economic growth and development of any country. The paper is concluded by considering some numerical example and discussing the effect of corruption.
**Keywords:** Corruption Precision Index, Magnitude of corruption, Government expenditure and revenue.
**AMS Classification numbers**: 39A10, 39A60, 65Q10

## 1. Introduction

Corruption scandals affect the economy of all countries. Corruption continues to pose a significant challenge throughout the globe. Transparency International (a major anti-corruption organization) defines corruption as Corruption is the abuse of entrusted power for private gain. Increasing corruption can act as a speed-breaker in the Indian growth story. The 2012 Transparency International Corruption Perceptions Index ranks India 94 out of 176 countries (100 very clean - 0 most corrupt) [3] indicating the severity of the issue.

We analyse the effect of corruption on government expenditure and revenue. The purpose of this paper is to show that output and growth are influenced by the level of corruption. We present a model that will be used to measure the effect of corruption on economic growth in India. The growth of economy has been considered as growth rate of GDP (gross domestic product) [11].The corruption

index in this paper is based on two studies one at international level and another at national level.

The model is being formulated by using difference equations.In this paper we will discuss how corruption affects -

- The Government expenditure
- Government Revenue

**1.1 Preliminaries:**
**Linear first order difference equation and its solution**

The linear first order difference equation has the form

$$f_0(k)y_{k+1} + f_1(k)y_k = g(k), \qquad k = 0, 1, 2, 3, ...$$

Over the indicated set of k-values, where the function $f_0$ and $f_1$ are non zero functions. If $f_0, f_1$ and g are constant functions. Then the equation can be written in the form $y_{k+1} = Ay_k + B$, where A & B are arbitrary constants
Its solution is

$$y_k = A^k y_0 + B(1 - A^k)/(1 - A), \quad \text{if } A \neq 1, \quad k = 0, 1, 2, ...$$

or

$$y_k = y_0 + Bk \quad \text{if} \quad A = 1$$

or

$$y_k = A^k y_0, \quad \text{if} \quad B = 0, \quad k = 0, 1, 2, ...$$

**2.1 Main Model**
**2.1.1 Construction of an equivalent mathematical model for Government expenditure:**

The mathematical study of the effect of corruption will be discussed by forming difference equations. Let $G_t$ be the Government expenditure at time t, $P_t$ be the revenue at time $t, g_t$ be GDP growth rate at time t, corruption precision index be $c_t$.

Let $G_{t+1}$ be Government expenditure at time $(t + 1), G_t$ be the government expenditure at time t. In developing nations to have growth in economy, the government expenditure increases it by $g_t G_t$. Here the growth rate g is being taken as the growth rate of GDP (gross domestic product). To maintain growth of economy, the government expenditure has to grow accordingly. Corruption will increase the government expenditure, say by $(c_t G_t)$. Then by the dynamics of

Government expenditure over time would be given by (including corruption affect). Here we consider $g_t$ and $c_t$ as constants.
If $g_t$ and $c_t$ are constant, then $g_t = g, c_t = c$

$$G_{t+1} = G_t + gG_t + cG_t$$

$$G_{t+1} = G_t + G_t(g + c)$$

$$G_{t+1} = G_t(1 + g + c)$$

This is a first order linear difference equation with constant coefficients and its general solution is given by

$$G_t = G_0(1 + g + c)^t, \tag{2.1.1}$$

If there is no corruption then $c = 0$ and therefore the government expenditure at time t, without corruption will be given by.

$$G_t^* = G_0(1 + g)^t, \tag{2.1.2}$$

Here we see that the effect of corruption by the increase in the Government expenditure over time. The ( Dent on Exchequer) is given by

$$B_x = (G_t - G_t^*) \tag{2.1.3}$$

This was the case when g and c are constants.

**2.1.2 Construction of an equivalent mathematical model on Effect of corruption on revenue generation:**

The government to maintain the economy of its country and for the upliftment of its people needs revenue which it gets from various sources. To generate revenue, government auctions its natural resources but at times this revenue gets dented due to corruption.

Let A be the initial revenue of the government and let $P_n$ be the revenue which it proposes to earn by at the end of nth year, to maintain the growth rate of the economy, the revenue should increase by its gdp. Corruption will decrease the revenue by corruption index.
If $g_t$ and $c_t$ are constants, $g_t = g, c_t = c$
Then $P_{n+1}$ the revenue at the end of $(n + 1)$th year is given by using difference equation .

$$P_{n+1} = P_n + gP_n - cP_n$$

or

$$P_{n+1} = P_n(1 + g - c).$$

This is a first order linear difference equation with constant coefficients and its general solution is given by

$$P_n = C(1 + g - c)^n$$

Where C is an arbitrary constant to be determined with the help of initial conditions.
If $P_n = P_o$, when n=0 then we have

$$C = P_0,$$

Then putting the value of C we have
$P_n = P_0(1 + g - c)^n$ here we have $P_0 = A$, therefore

$$P_n = A(1 + g - c)^n, \tag{2.2.1}$$

This is the revenue of the government at time n with corruption.
Let $P_n$ be the revenue of the government at time n without corruption , $c = 0$, then

$$P_n^* = A(1 + g)^n, \tag{2.2.2}$$

The dent to the exchequer is given by

$$B_i = (P_n^* - P_n) \tag{2.2.3}$$

The corruption index in this paper is based on two studies one at international level (CPI) and another at national level (magnitude of corruption). When we discuss Governments involvement through FDI then we can consider corruption index at international level and if we study some scam at national level then we can take corruption index at national level. But if corruption at both levels are involved then we take combination of both indices. For example FDI in defence then corruption at both levels can be considered.

Here we construct different models by constructing corruption index in different ways. The corruption index is being estimated in many ways, depending upon level of the corruption and its effect. Corruption Index is directly proportional to (CPI), [12] or (magnitude of corruption) [3] or mean of both. Here we have taken two types of means, Corruption Index = constant* (CPI) where (0<constant<1), Corruption Index < g (GDP).

**Model 1:**

In our first model we will form corruption index as follows:
Corruption Index is directly proportional to (CPI), or (magnitude of corruption).
Either c=constant*(CPI), or c=constant*(magnitude of corruption).

The constant can be taken in many ways, either 1/2 or 1/5 or 1/10 or 1/20 or 1/40 or 1/50 depending upon the level of the corruption and its effect.

If CPI is $(1 - 88/176) = 0.5$, then $c = (.5)/10 = .05$, or $c = (0.5)/50$ or if magnitude of corruption is 40 % or (40/100), then $c = (0.4)/10 = 0.04$ or $c = (.4)/40 = .01$.

**Model 2:**

In our second model we will form corruption index as follows:

Corruption Index is directly proportional to Arithmetic mean of (CPI) and (magnitude of corruption). Then c = constant x [(CPI) + (magnitude of corruption)] /2

The constant can be taken in many ways, either 1/2 or 1/5 or 1/10 or 1/40 or 1/50 or 1/100 depending upon severity of the corruption and its effect.

If CPI is $(1-88/176) = 0.5$ and if magnitude of corruption is 40 % or (40/100), then
$c = (1/10) \times [0.5 + 0.4]/2 = (0.045)$

**Model 3:**

In our third model we will form corruption index as follows:

Corruption Index is directly proportional to Geometric mean of (CPI) and (magnitude of corruption).
Then c = constant x Sqrt [(CPI) x magnitude of corruption)] .
The constant can be taken in many ways, either 1/10 or 1/40 or 1/50 or 1/100 depending upon severity of the corruption and its effect.

If CPI is 0.5 and if magnitude of corruption is 40% or (40/100), then
$c = (1/10) \times Sqrt[0.5x0.4] = (0.0447)$.

To make the mathematical approach more realistic we have considered data from the latest survey done by E & Y [3] on people from various fields. Their some of the specific findings on corruption are:

| **S.No.** | **Sectors perceived to be most Corrupt (more than** 50%**)** | **Sectors perceived to be Corrupt** (25% − 50%) | **Sectors perceived to be less Corrupt (less than** 25%**)** |
|---|---|---|---|
| 1 | Infrastructure & real estate (85%) | Telecom | Pharmaceutical |
| 2 | Metals & mining (76%) | Oil & Gas | Retail & Consumer Products |
| 3 | Aerospace & defence (64%) | Technology, Media & Entertainment | Manufacturing |
| 4 | Power & utilities sectors(51%) | Financial Services | – |

Here in this paper we will measure the effect of corruption, by considering these factors corruption precision index(CPI) as given by transparency international as (1-94/176),magnitude of corruption as number of people affected by it say, as by table above the magnitude of corruption in financial sector is 25% or(25/100). According to a recently released biannual country update by the World Bank, the overall outlook for India remains positive with its economic growth expected to accelerate FY2016 . We will consider that the growth rate in expenditure will be according to growth in GDP (gross domestic product) according to ministry of finance the GDP growth rate in 2010 was 5.1%.

**Results & Discussions**

**Numerical Example:**

**Government expenditure:**

The calculation will be done by taking Government expenditure $G_t$ =100000 (CPI) /50 = (.5) / 10= .05 = c, or c can be taken as (magnitude of corruption)/10 = (.4)/1 = .04, Growth rate = GDP=g=.05.

Putting these values in equations (2.1.1), (2.1.2) and(2.1.3) we get

$$G_t = 100000(1 + .05 + .05)^t$$

$$G_t^* = 100000(1 + .05)^t$$

$$B_x = G_t - G_t^*$$

**Government Revenue**

The calculation will be done by taking Government revenue $P_n$ =100000 (CPI) /50 = (.5) /10 = .05 = c, or c can be taken as (magnitude of corruption)/10 = (.4)/1 = .04, Growth rate = GDP=g=.05.

Putting these values in equations (2.2.1),(2.2.2) and(2.2.3) we get

$$P_n = 100000(1 + .05 + .05)^n$$

$$P_n^* = 100000(1 + .05)^n$$

$$B_i = P_n - P_n^*$$

**Forms of corruption: International level-corruption precision index (CPI) / National level-Magnitude of corruption.**

**Table for Government expenditure and Effect of corruption**

Initial Government expenditure = 1, 00,000, Time period=3years, GDP=g=.05

| **Model** | **Government expenditure without corruption** $G_t*$ | **Forms of corruption: corruption precision index (CPI) / Magnitude of corruption** | **Government expenditure with corruption** $G_t$ | **Effect of corruption-Dent on Exchequer** $B_x = G_t - G_t*$ |
|---|---|---|---|---|
| Model-1 | 116075.45 | 0.05/0.04 | 1134488.88 /130605.00 | 18413.43 /13529.55 |
| Model-2 | 116075.45 | (.05+.04)/2 | 132533.00 | 16457.55 |
| Model-3 | 116075.45 | Sqrt(.05x.04) | 132417.33 | 16341.88 |

**The Table for Government Revenue and Effect of corruption.**

Initial Government Investment = 1,00,000 , Time period=3years , GDP=g=.06

| **Model** | **Government Revenue without corruption Pn*** | **Forms of corruption: corruption precision index (CPI) / Magnitude of corruption** | **Government Revenue with corruption Pn** | **Effect of corruption-Dent on Exchequer Bi= Pn*-Pn** |
|---|---|---|---|---|
| Model-1 | 119561.82 | 0.05/0.04 | 103041.60 /106167.78 | 16520.22 /13394.04 |
| Model-2 | 119561.82 | (.05+.04)/2 | 104593.98 | 14667.84 |
| Model-3 | 119561.82 | Sqrt(.05x.04) | 104687.80 | 14874.02 |

The increase in government expenditure and decrease in government revenue are direct affect of corruption on Indian economy which will lower the growth of the country. For the development of the country the corruption has to be brought down and gradually be eradicated.

## 4. Conclusion

Measuring corruption requires a model that pays special attention to the kind and level of corruption. Moreover, accurate data are needed to estimate any sophisticated model assessing the level of corruption. In this paper, an effort has been made to show the effect of corruption on Indian economy by estimating the corruption by corruption index and forming a model of corruption, using difference equations.

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